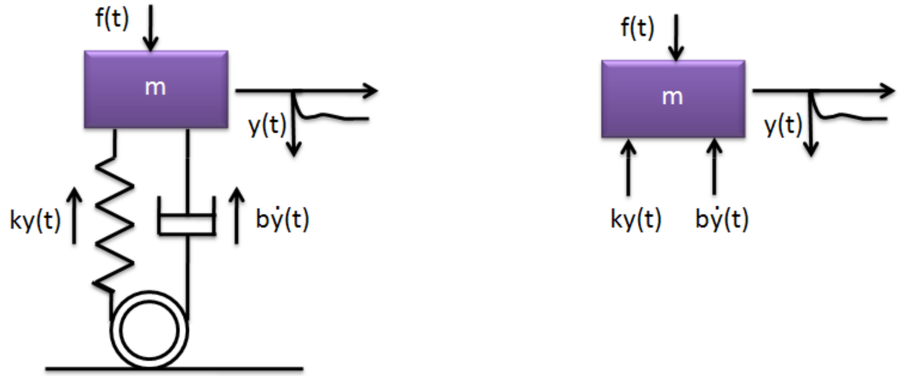
3. Transfer Function

EN2142 Electronic Control Systems

Dr. Rohan Munasinghe
Department of Electronic and Telecommunication Engineering
Faculty of Engineering
University of Moratuwa 10400

Shock-Absorber Model



$$\ddot{y}(t) + 2\sigma\dot{y}(t) + \rho y(t) = \eta f(t)$$

$$2\sigma = \frac{b}{m},\ \rho = \frac{k}{m},\ \text{and}\ \eta = \frac{1}{m}$$

Shock-Absorber Response

$$s^2Y(s) - sy(0) - y'(0) + 2\sigma[sY(s) - y(0)] + \rho Y(s) = \eta F(s)$$
$$(s^2 + 2\sigma s + \rho)Y(s) - y(0)s - [2\sigma y(0) + y'(0)] = \eta F(s)$$

$$Y(s) = \frac{y(0)s + [2\sigma y(0) + y'(0)]}{(s^2 + 2\sigma s + \rho)} + \frac{\eta}{(s^2 + 2\sigma s + \rho)}F(s) \qquad (3.52)$$

Shock-Absorber Transfer Function

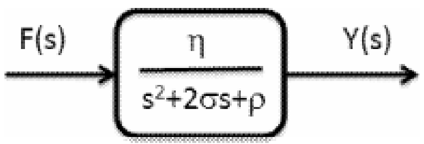
Neglect the Initial condition response

Then

$$Y(s) = \frac{\eta}{s^2 + 2\sigma s + \rho}F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{\eta}{s^2 + 2\sigma s + \rho}$$

$$G(s) = \frac{\eta}{s^2 + 2\sigma s + \rho}$$



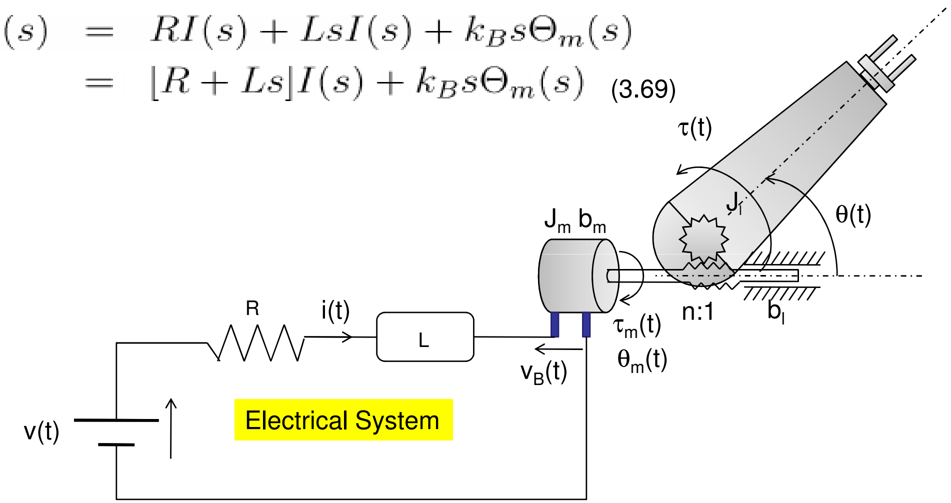
Example: Robot Link Motion

- Electrical System

$$v(t) = iR + L\frac{di(t)}{dt} + v_B(t) \qquad v_B(t) = k_B\dot{(\theta)}_m$$

$$v(t) = iR + L\frac{di(t)}{dt} + k_B\dot{\theta}_m(t)$$

$$V(s) = RI(s) + LsI(s) + k_Bs\Theta_m(s)$$
$$= [R + Ls]I(s) + k_Bs\Theta_m(s) \qquad (3.69)$$



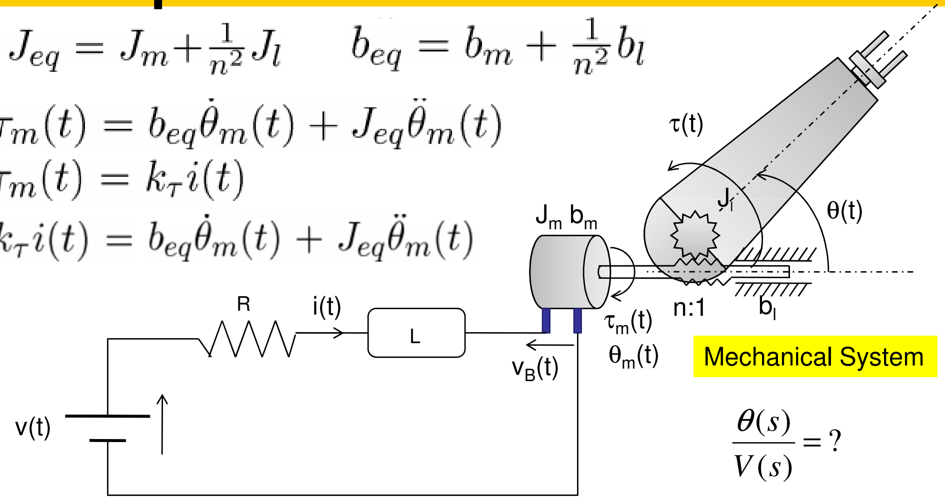
Example: Robot Link Motion Control

$$J_{eq} = J_m + \frac{1}{n^2} J_l \qquad b_{eq} = b_m + \frac{1}{n^2} b_l$$

$$\tau_m(t) = b_{eq}\dot{\theta}_m(t) + J_{eq}\ddot{\theta}_m(t)$$

$$\tau_m(t) = k_\tau i(t)$$

$$k_\tau i(t) = b_{eq}\dot{\theta}_m(t) + J_{eq}\ddot{\theta}_m(t)$$



$$\frac{\theta(s)}{V(s)} = ?$$

$$I(s) = \frac{1}{k_\tau}\left[J_{eq}s^2\Theta_m(s) + b_{eq}s\Theta_m(s)\right]$$

$$= \frac{1}{k_\tau}\left[J_{eq}s + b_{eq}\right]s\Theta_m(s) \qquad (3.70)$$

(3.70) into (3.69)

$$V(s) = \frac{1}{k_\tau}(R+Ls)(J_{eq}s+b_{eq})s\Theta_m(s) + k_B s\Theta_m(s)$$

$$k_\tau V(s) = \left[(R+Ls)(J_{eq}s+b_{eq}) + k_\tau k_B\right]s\Theta_m(s)$$

$$\frac{\Theta_m(s)}{V(s)} = \frac{k_\tau}{\left[(R+Ls)(J_{eq}s+b_{eq}) + k_\tau k_B\right]s}$$

$$\theta_m(t) = n\theta(t)$$

$$\frac{\Theta(s)}{V(s)} - \frac{k_\tau/n}{\left[(R+Ls)(J_{eq}s+b_{eq}) + k_\tau k_B s\right]s}$$

$$= \frac{k_\tau/n}{LJ_{eq}s^3 + (Lb_{eq}+RJ_{eq})s^2 + (Rb_{eq}+k_Bk_\tau)s} \qquad (3.73)$$

- System Transfer Function -

Variables and Parameters

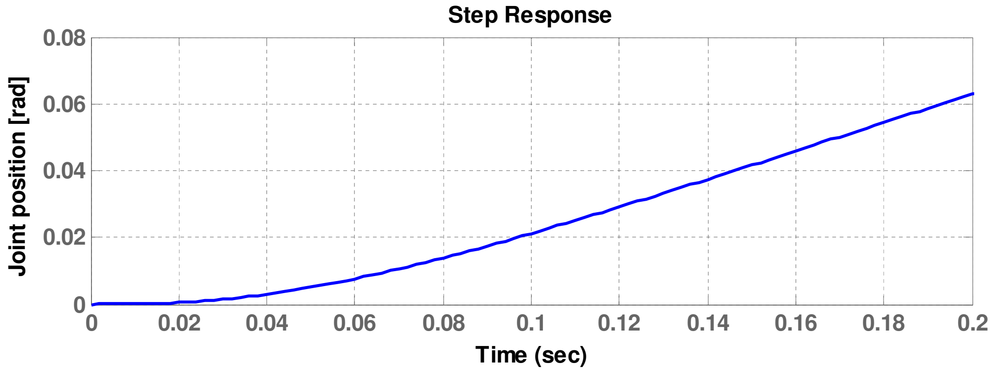
$v(t)$: armature voltage[V]	$i(t)$: armature current[A]
J_m: motor inertia [Kgm^2]	k_τ: torque constant [Nm/A]
n: gear ration	τ_l: load shaft torque [Kgm^2]
L: armature inductance [Vs/A]	$v_B(t)$: back electromotive force [V]
b_m: motor viscous damping constant [Nms/rad]	$\dot{\theta}_m(t)$: motor shaft speed [rad/s]
J_l: arm inertia [kgm^2]	R: armature resistance [Ω]
k_B: motor back emf constant [Vs/rad]	τ_m: motor shaft torque [Nm]
$\theta_m(t)$:motor shaft position [rad]	b_l: viscous damping constant of the arm [Nms/rad]
$\theta(t)$: arm position [rad]	

MatLab Simulation

```
L = 0.062;
R = 2.5;
n = 20;

kt = 0.026; %Nm/A
kb = 0.02;  %V/rad.s-1

Jeq = 0.00004; %kg/m2
beq = 0.001; %Nm/rad.s-1

b2 = L*Jeq;
b1 = L*beq+R*Jeq;
b0 = R*beq+kb*kt;
a0 = kt/n;

sys=tf([a0],[b2 b1 b0 0]);
step(sys); grid on;
```

Unit step (1V) response

Step Response of Robot Link Position



Position Response => Velocity Response

Position

$$\frac{\Theta(s)}{V(s)} = \frac{k_\tau/n}{[(R+Ls)(J_{eq}s+b_{eq})+k_\tau k_B s]s} = \frac{k_\tau/n}{LJ_{eq}s^3+(Lb_{eq}+RJ_{eq})s^2+(Rb_{eq}+k_B k_\tau)s}$$

Speed

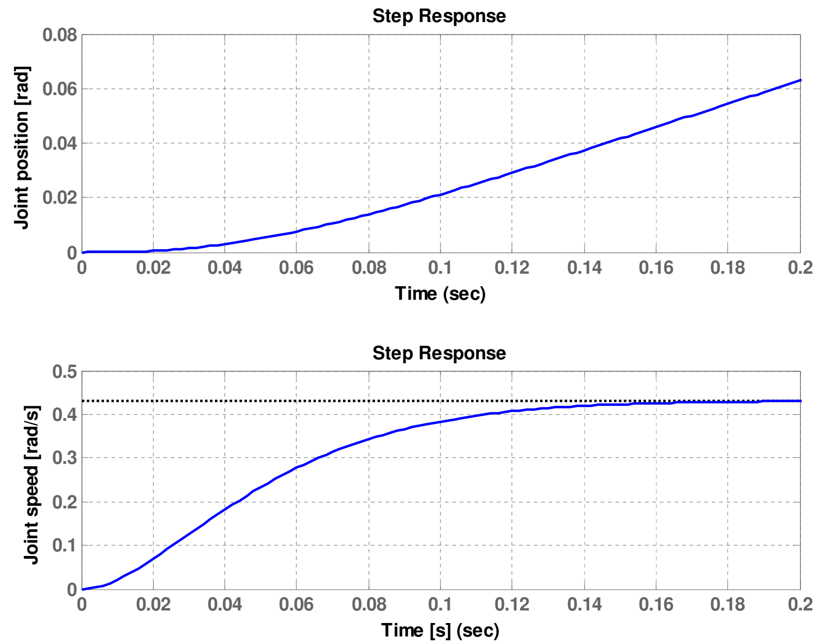
$$\frac{s\Theta(s)}{V(s)} = \frac{k_\tau/n}{LJ_{eq}s^2+(Lb_{eq}+RJ_{eq})s+(Rb_{eq}+k_B k_\tau)} \quad (3.73)$$

Robot Link Velocity Response

```
L = 0.062;
R = 2.5;
n = 20;
dur=0.2;

kt = 0.026; %Nm/A
kb = 0.02;  %V/rad.s-1

Jeq = 0.00004; %kg/m2
beq = 0.001; %Nm/rad.s-1

b2 = L*Jeq;
b1 = L*beq+R*Jeq;
b0 = R*beq+kb*kt;
a0 = kt/n;

Psys=tf([a0],[b2 b1 b0 0]);
subplot(211); step(Psys,dur); ylabel('Joint position [rad]'); grid on;

Vsys=tf([a0],[b2 b1 b0]);
subplot(212); step(Vsys,dur); ylabel('Joint speed [rad/s]'); grid on;
xlabel('Time [s]')
```

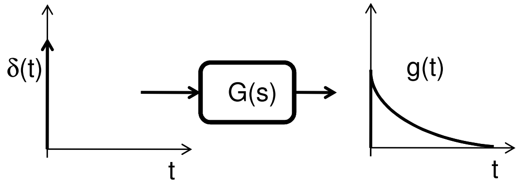
Link Position and Velocity Response

1V input



3.31 Determination of Tr Fn

- Give a unit impulse input *δ(t)*, record the unit impulse responses *g(t)*



- Derive the Laplace Transform of *g(t)*

$$Y(s) = G(s)R(s) \quad r(t) = \delta(t) \Rightarrow R(s) = 1$$

$$Y(s)|_{R(s)=1} = G(s)$$

$$G(s) = \mathcal{L}\{g(t)\}$$